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Controllability Analysis of Multi Objective Control Systems.

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Abstract— **The performance requirements stated in project specifications often comprise conflicting objectives. These objectives may further be a complex mix of steady state and dynamic performance. Control devices such as solenoid actuators are often chosen purely on steady state force characteristics, due to the difficulty of appraising the conflicting and generally non-linear nature of the performance objectives. This can have ramifications in terms not only of the actuator performance, but also in the overall controllability of the system when closed-loop control is implemented. An example automotive application examining the multi objective controllability of electronically actuated valves is presented. Multi Objective Evolutionary techniques are utilised to derive the optimal force-displacement characteristics and also dynamic characteristics of the desired actuator under the constraint of design performance criteria. The selected actuator is then assessed for its controllability and dynamic performance.**

Keywords— **Solenoid, Multi Objective Design, Optimisation, Controllability, Automotive.**

I. Introduction

The controllability of an actuator is often considered relatively late in the process of project development. Industrial partners often co-operate in the definition of performance criteria by which candidate mechanical designs are developed. Subsequent selection of control hardware to achieve the defined performance criteria is then often made in terms of static force characteristics, after which time the control developer must implement control algorithms to achieve the desired real time dynamic performance. In order to develop a structured approach to project development, an automotive control application is presented in order to investigate the problem. The application focusses on the control of normal-force solenoid actuators in the context of strict performance criteria, although it will be shown that the technique can form the basis of a more generic approach. The aim of the multi objective design approach presented in this paper is to design the physical system in such a way that the control engineer implements the control algorithms on a platform which can achieve the dynamic performance required by the design specifications, while staying within other design implementation bounds such as current and voltage limitations. Two approaches are examined. Firstly, a suitable candidate force-displacement profile which would allow the mechanical system to comply with the performance criteria will be found using multiobjective evolutionary techniques. This makes available to the electromagnetic design engineer a far more detailed force profile than simply providing a maximum force requirement. This approach does not consider or include the dynamic limitations imposed by the behaviour of current, flux and force in the actuator. A candidate electromechanical actuator will thus be examined by a similar multiobjective approach, this time to confirm its controllability in the context of the project dynamic performance criteria. It will be shown that the application of multiobjective techniques can contribute to the design of control systems which are controllable in the sense of extremely complex and in some cases conflicting design criteria.

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II. Problem Definition

An example actuator will be presented to exemplify the application of the multi objective approach. A real-life automotive case study will be investigated to test the applicability of the technique. An electronically actuated valve system is considered (figure 1), with balanced opposing springs (to minimise the reactive power driven through the power electronics) of equal rate, a full stroke of $8mm$, equilibrium position at $4mm$, and opposing solenoids to effect control over the movement of the armature which bears on the valve stem. The two springs exchange potential energy during the motion of the valve between the two operating points (fully open and fully closed). The actuators compensate for the energy losses during motion, and also fulfill the requirement to hold the valve at the operating points. The controllability of the system is evaluated in the context of the project performance requirements, namely (Wang et.al, 2000);

• Transition time from fully open to fully closed must be less than $3.5ms$.
• To minimise acoustic noise, landing velocity to the fully closed position must be less than $0.05m/s$.
• To minimise acoustic noise, the closing of the valve clearance (the clearance between the armature and the valve stem to ensure positive closing, otherwise known as lash) from the fully open position must be less than $0.05m/s$.
• To minimise acoustic noise, landing velocity to the fully open position must be less than $0.05m/s$.
• At a supply voltage of $42V$, the maximum catching current must typically be $40A$.

• At a supply voltage of $42V$, the maximum holding current must typically be $7.5A$.

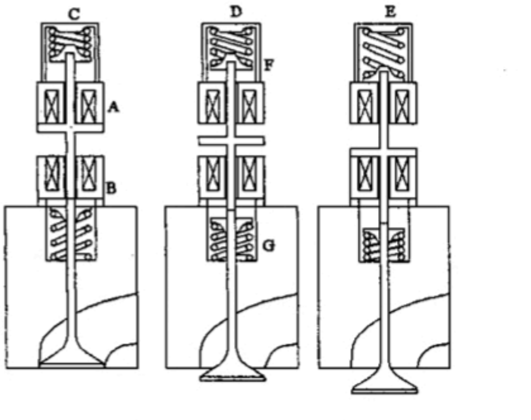

Fig. 1. Electronically controlled engine valve actuator. A upper solenoid, B lower solenoid, C fully closed, D equilibrium position, E fully open, F upper spring, G lower spring.

The control algorithm must be able to dynamically open and close the valve subject to these operational constraints. It can be seen that a set of position-velocity trajectories exist to satisfy these requirements, however the choice of actuator to achieve this performance is an extremely complex one, compounded by the inherently nonlinear force capabilities of solenoid actuators. Although controllability analysis has been applied to nonlinear systems [Hermann and Krener 1977], [Nijmeijer, 1982], [Schaft], a point is reached where the performance requirements of the system in conjunction with all the other lumped system nonlinearities and discontinuities becomes to difficult a task for standard analysis. Also, the mathematical descriptions involved become too complex for the engineer to analyse. Evolutionary algorithms have been applied to a variety of control system design problems [Chipperfield and Fleming, 1996], [Fonseca and Fleming, 1998]. More specifically, genetic algorithms have been used [Zaoui and Marchand, 1998] to optimize two dimensional finite element electromagnetic design of normal force solenoid actuators. This was however a single objective approach to optimize the force displacement profile over a range of airgaps. In the approach under consideration here, the evolutionary algorithms will perform a multi objective search for the Pareto-Optimal set of candidate actuators which fulfill the cost function of controllability. The design will proceed as follows: The dynamic force-displacement characteristic required of the mechanical system will be analysed, an actuator topology will be selected, and finally the actuator controllability will be confirmed. An experimentally verified mechanical model in Simulink is used for performance verification, and also as the platform for the multi objective searches.

III. Force-Displacement Optimisation

The mechanical valve system can be modelled (figure 2) as [Wang et.al.,];

$$\ddot{y} = \frac{F_1 - F_2 - B\dot{y} - 2Ky}{m} \tag{1}$$

where y is the position of the armature relative to the equilibrium position, F_1 and F_2 are the lower and upper actuator forces respectively, B is the friction constant, K is the effective spring rate of the pair of springs, and m the effective total mobile mass. We shall for the purposes of analysis consider the transition from valve fully open to valve fully closed, and formulate an objective function to be assessed by the Multi Objective Genetic Algorithm (MOGA). The objective function is articulated as follows;

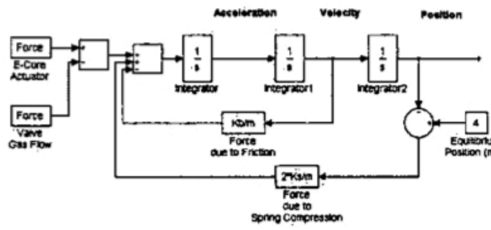

Fig. 2. Mechanical model.

• Objective 1: Minimise the transition time.
• Objective 2: Minimise the integral of force with respect to time from the respective actuators.
• Objective 3: Minimise applied force at large airgaps.
• Objective 4: Minimise the maximum applied force value.
• Objective 5: Landing speed must be less than $0.05m/s$.
• Objective 6: On release, the maximum armature velocity closing the valve gap must be less than $0.05m/s$
• Objective 7: Minimise the integral of force derivative with respect to time.

Objectives 1,5 and 6 are obvious according to the design criteria, however, the other objectives require some clarification. Objective 2 seeks to minimise the overall power consumption of the actuators, since the force developed by the actuators will be related to current, with consequent i^2r losses in the windings. Objective 3 reflects the typical solenoid force displacement characteristic which can be approximated as:

$$F = \frac{K_f(i)^2}{e^2} \tag{2}$$

where K_f is the solenoid force constant, i is the current and e is the airgap. Consequently, solutions which require force at large airgaps are heavily penalised. Objective 4 seeks to minimise the overall maximum value of the applied force, again to reduce the effects of losses. Finally, Objective 7 seeks to find the smoothest force profile in order to maximise the potential for controllability of the actuators, since it will be seen that force derivatives at small airgaps are a severe limiting factor to the dynamic performance of solenoid actuators.

The GA Toolbox for Matlab with the MOGA extension tools developed at the University of Sheffield was utilised to perform the simulation routines. MOGAs have routinely been applied to control optimisation problems such as gain scheduling or controller parameter optimisation [Schroder et.al., 2001], [Griffin et.al., 2000]. However in this case, we are trying to identify the optimal dynamic force-displacement characteristic to operate within the bounds set by the design criteria. The decision variables are in this case assigned to a quantised map of the position of the armature, with variables clustered more closely at small airgaps, since we are anticipating the bulk of the control action to occur at airgaps less than $1mm$. The parameters associated with the MOGA setup were as follows;

- Population size: 100
- Number of decision variables: 27
- Number of objectives: 7
- Number of immigrants per generation: 6
- Coding: Gray, 20 bits per decision variable, except where varied [Zitzler et.al 2000]
- Selection: Stochastic universal sampling [Baker, 1987]
- Recombination: Single-point binary crossover, probability =0.7
- Mutation: Element-wise bit-flipping, expectation of 1 bit per chromosome
- Generational gap: Zero
- Random injection: 2 random chromosomes per generation
- Elitism: None
- Fitness assignment: [Fonseca and Fleming, 1993] multiobjective ranking. Transformation from rank to fitness using linear fitness assignment with rank-wise averaging.
- External population: Off-line storage of nondominated solutions
- Fitness Sharing: (Parameter-less) Epanechnikov fitness sharing [Fonseca and Fleming, 1995] Implemented in criterion space
- Mating restriction implemented: distance set to the niche size parameter found by the Epanechnikov sharing algorithm

Applying the multiobjective algorithm to the mechanical system results in a velocity-position profile which satisfies the project design requirements (figure 3) The motion of the armature under the candidate force-displacement profile (figure 4) satisfies all the soft release, soft landing and transition time criteria from the project definition, it also inherently possesses a relatively smooth force trajectory, which should also represent one of the lowest power consumptions when followed by a pair of actuators. This analysis considerably increases the detail of the design requirements which

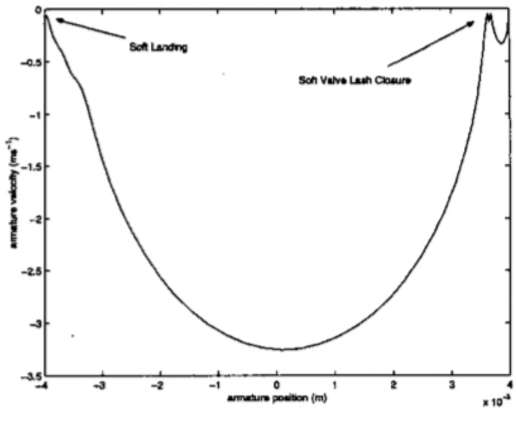

Fig. 3. Optimised position-velocity profile.

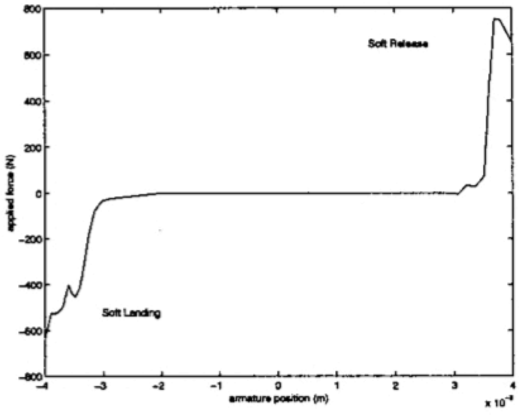

Fig. 4. Candidate force-displacement profile.

can be supplied to the electromagnetic design engineer. However, a limiting factor in the operation of solenoids at relatively small airgaps is due to the current and flux dynamics. An actuator candidate will now be considered to assess its performance suitability to achieve the required dynamics, using information derived from finite element analysis.

IV. Actuator Multiobjective Controllability Analysis

Assuming a given valve pitch for the particular cylinder head under examination, a solenoid actuator can be designed and simulated using finite element methods [Simkin and Trowbridge, 1991], [Ratnajeevan and Hoole, 1989] based around a core size bounded by the valve pitch size, current and voltage limitations and other dimensional limitations of the cylinder head. By way of example, applying a magneto-static approach (force analysis at fixed airgaps), a force-current-displacement map for a particular design can be derived (figure 5) comparison with the optimized force-displacement requirement (figure 4) confirms the suit-

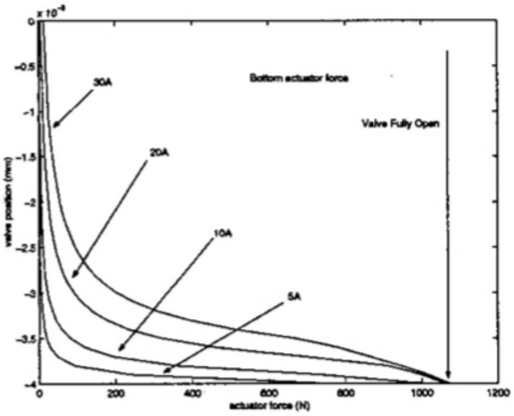

Fig. 5. Solenoid actuator static force-current-displacement characteristic

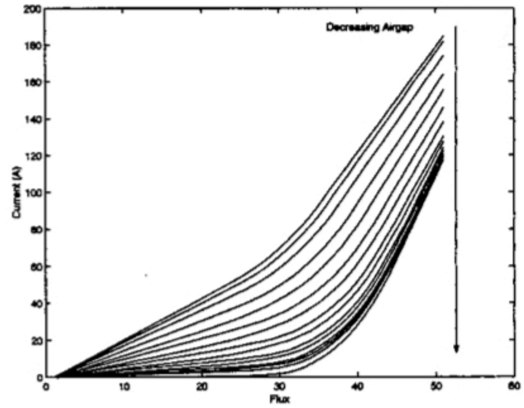

Fig. 7. Actuator flux-current-displacement map from finite element analysis

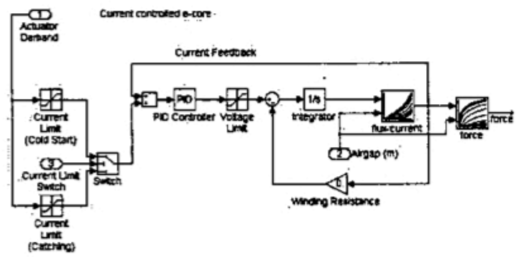

Fig. 8. Electromagnetic model for controllability analysis

ability of the candidate actuator in steady-state terms, however the controllability of the actuator in dynamic, operational terms cannot be confirmed. An electromagnetic dynamic model approximation is developed to allow multiobjective analysis to again be applied. This time the analysis will seek to confirm the controllability of the actuator, that is whether a control action trajectory exists which satisfies all the dynamic requirements and constraints. In an iterative environment, further design steps take place if the design cannot satisfy the project requirements.

The relationships between flux, current and force ascertained by finite element analysis and can thus be im-

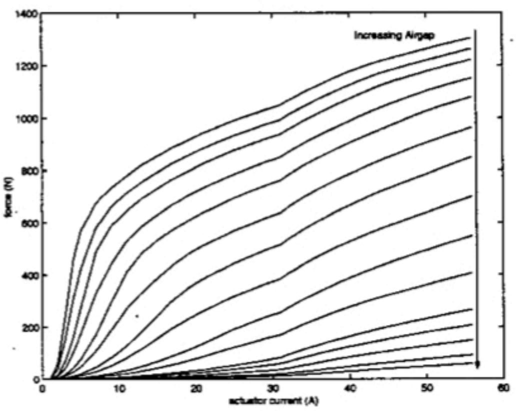

Fig. 6. Actuator force-current-displacement map from finite element analysis

plemented into a Simulink model (figure 8) The model is coupled to the mechanical system and provides a platform on which to postulate the question "is this system controllable in the sense of performance requirements and design constraints?" The system is controllable in this sense if an applied current trajectory exists which when applied to the actuators, causes the armature to move in a position-velocity path which satisfies the mechanical constraints of the application (soft landing etc.). The current trajectory must also comply with the maximum current constraint, and voltage must be limited in the simulation to the application dc link voltage. To achieve this end, a multiobjective analysis is constructed with the same quantisation of 27 decision variables as before. In this case, the decision variables provide a position-current demand vector to be tracked by the electromechanical model (figure 8. Tracking is achieved by a PID current controller, with the Multiobjective evolutionary algorithm set to the same operational parameters and objective function as before (with "force" replaced by "current". Also the number of objectives was reduced to 6 by omitting the overall max current objective to increase the computation time. It had been found that the force-distance penalties performed the same function.

V. Results

Figure 9 is a non dominated tradeoff output from the multiobjective analysis. The objectives to be minimised are numbered as follows;

- 1. Actuator transition time
- 2. Bottom actuator $distance^2 * current$
- 3. Soft landing velocity
- 4. Current derivative

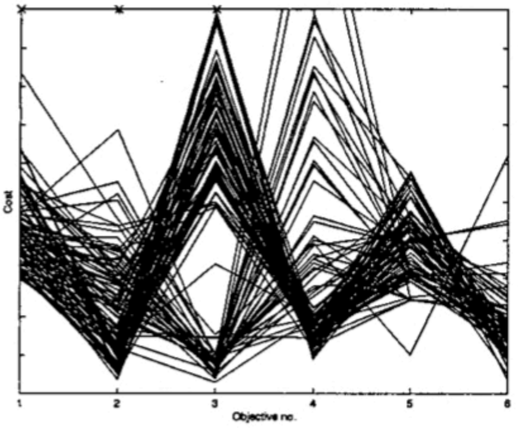

Fig. 9. Non dominated tradeoff graph for the actuator controllability analysis

- 5. Soft release (valve gap closing) velocity
- 6. Top actuator $distance^2 * current$

It is immediately apparent that the controllability of the system is dependent on conflicting objectives, revealing the salient operating principles of the constrained system. For example;

- The actuator transition time (1) can be reduced by relaxing the constraints on currents applied at large airgaps (2), however this incurs a cost in terms of the soft landing (3).
- Soft landing (3) and soft release (5) incur a cost in terms of the current derivatives (4).
- Close examination of the tradeoff graph reveals a particularly concerning tradeoff, that is there is only a small number of candidate solutions in which both soft release and soft landing are achieved.

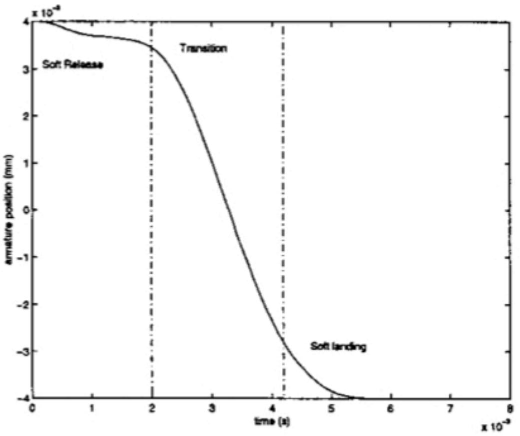

Fig. 10. Successful time-position trajectory

The single successful solution selected does comply with all the required dynamic project requirements in terms of soft landing, soft departure, and transition time (figures 10, 11). The constraint of $40A$ maximum current was however found in all cases impossible to comply with to achieve the required dynamic performance. Although the $42V$ supply voltage was strictly adhered to, it was found that the maximum current constraint needed to be relaxed in order to achieve the performance requirements. With a current limit of $45A$, then a current trajectory does exist, which when tracked by standard PID current control loops produces a position velocity trajectory for the armature which complies with the required dynamic performance. A revised current limit has also been identified which allows these objectives to be achieved. The approach has identified a means to confirming the controllability of complex nonlinear systems and is worthy of further development.

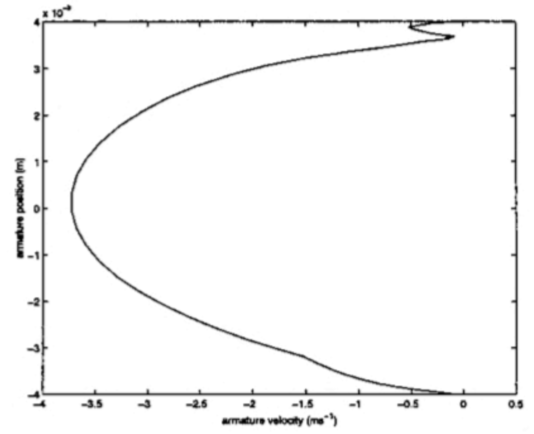

Fig. 11. Successful velocity-position trajectory

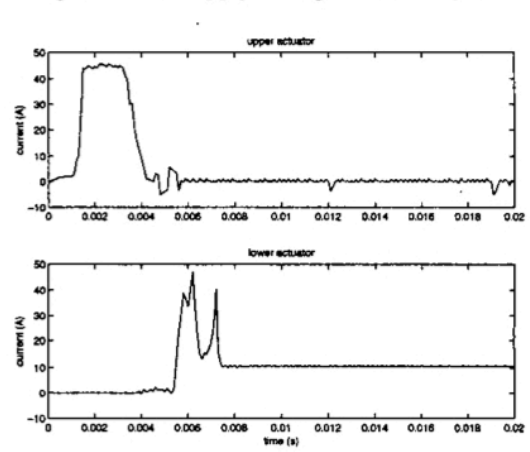

Fig. 12. Upper and lower actuator current waveforms

VI. Conclusions

An application example has been presented in order to illustrate a technique to answer the question "is the system controllable?". Although the example investigates an existing actuator candidate, it is envisaged that the technique will exist as part of a greater multiobjective design process. It has been shown that in the example, the actuator as designed, does satisfy the static force requirements and thus should be capable of effecting adequate operation of the armature. However in practice the control design to achieve all the project performance objectives was proving difficult to achieve. In general the multiobjective controllability analysis would have been performed before the manufacture of experimental actuators, and an iterative process performed with finite element analysis to achieve a controllable system. In the case presented here, we can analyse the existing design, and enter an iterative design phase to produce an improved actuator. It can also be seen that the technique has potential benefits to a wide range of applications where controllability analysis proves difficult or impossible by conventional methods.

It has been shown that the analysis has identified the requirement to increase the operational current limits in order to achieve controllability. It has also been shown that the group of successful candidate current vector solutions is extremely small. This is an area of the technique deserving further research. It would certainly verify the experimental problems which have been experienced implementing successful control design on the system. A conclusion which appears to be valid would be that the multivariable controllability analysis of the system also reveals a subset of potential robustness in any designed control systems. The small number of candidate solutions shows that the family of current vectors which can achieve controllability is small, and indeed narrow. This is confirmed by experimental experience which suggests the system is extremely susceptible to physical parameter variations.

The findings would suggest that the system is indeed controllable, however further iterations of the design procedure is necessary. This conclusion stems from the narrow band of successful solutions, suggesting an extremely sensitive system, and coupled to this the relationship between the current demand derivatives and soft release/landing. It was found in the search for successful candidate solutions that the restrictions on the current demand vector derivatives needed to be relaxed to achieve controllability, suggesting a significant limitation to controllability. After a narrow successful band, further relaxation produces no further successful candidates. This suggests that the time constants of current and flux (and hence force production) are a limiting factor at small airgaps, and thus the system would benefit in both controllability and robustness from further iterative design steps in terms of the actuator design. This process is currently in progress.

The process has successfully identified limitations and potential solutions to achieve controllability of a nonlinear constrained multiobjective system. Although controllability has been confirmed, the method has also confirmed experimental experiences regarding the difficulty of implementing successful control design. Although the technique is relatively new, its potential benefits are apparent for developing new systems, as is the potential for identifying controllability issues with existing systems.

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